Oxford Cambridge and RSA

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics
Question Paper

Wednesday 16 May 2018 – Morning
Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

Model Answers

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

1 Write $\frac{8}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$, where a and b are integers to be found. **[2]**

1. $\frac{8}{3-\sqrt{5}} = \frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{24+8\sqrt{5}}{9-5}$

$= \frac{24+8\sqrt{5}}{4}$

$= 6+2\sqrt{5}$

2 Find the binomial expansion of $(3-2x)^3$. **[4]**

2. $(3-2x)^3 = \binom{3}{0}(-2x)^0(3)^3 + \binom{3}{1}(-2x)(3)^2 + \binom{3}{2}(-2x)^2(3) + \binom{3}{3}(-2x)^3(3)^0$

$= 27 + 3(-2)9x + 3(4)(3)x^2 + (-8)x^3$

$= 27 - 54x + 36x^2 - 8x^3$

3 A particle is in equilibrium under the action of three forces in newtons given by

$$\mathbf{F}_1 = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \quad \mathbf{F}_2 = \begin{pmatrix} 2a \\ -3a \end{pmatrix} \quad \text{and} \quad \mathbf{F}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix}.$$

Find the values of the constants a and b. **[3]**

3. $\begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ -3a \end{pmatrix} + \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$8 + 2a = 0$

$2a = -8$

$a = -4$

$-3a + b = 0$

$b = 3(-4)$

$b = -12$

4 Fig. 4 shows a block of mass $4m$ kg and a particle of mass m kg connected by a light inextensible string passing over a smooth pulley. The block is on a horizontal table, and the particle hangs freely. The part of the string between the pulley and the block is horizontal. The block slides towards the pulley and the particle descends. In this motion, the friction force between the table and the block is $\frac{1}{2}mg$ N.

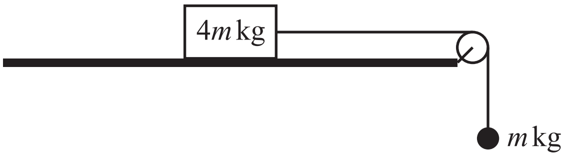
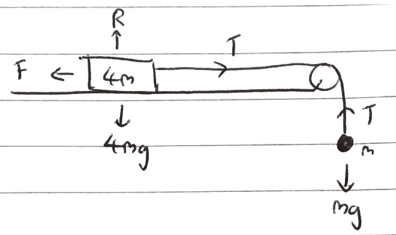


Fig. 4

Find expressions for

- the acceleration of the system,
- the tension in the string. **[4]**

4.



The two equations of motion are:

$mg - T = ma$

$T = mg - ma$ ①

$T - F = 4ma$

$T - \frac{1}{2}mg = 4ma$ ②

Sub ① into ②

$mg - ma - \frac{1}{2}mg = 4ma$

$\frac{1}{2}g = 5a$

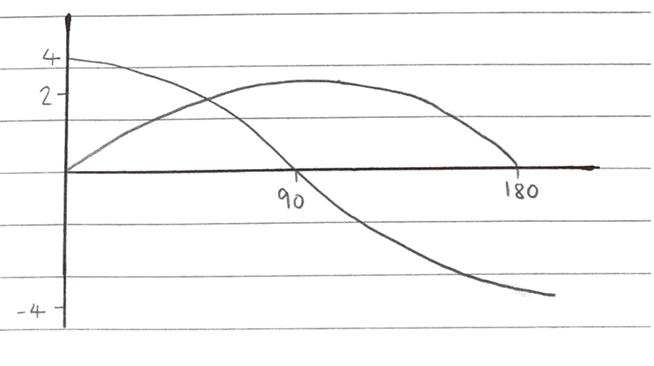
$a = \frac{1}{10}g \text{ ms}^{-2}$

$T = mg - m\left(\frac{1}{10}g\right)$

$T = \frac{9}{10}gm$ N

5 **(i)** Sketch the graphs of $y = 4\cos x$ and $y = 2\sin x$ for $0° \leqslant x \leqslant 180°$ on the same axes. **[2]**

5 i)

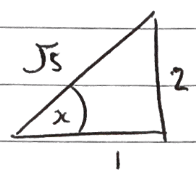


(ii) Find the exact coordinates of the point of intersection of these graphs, giving your answer in the form $(\arctan a, k\sqrt{b})$, where a and b are integers and k is rational. **[4]**

ii) $4\cos x = 2\sin x$

$2 = \tan x$

$x = \arctan 2$



$\cos x = \frac{1}{\sqrt{5}}$, $\sin x = \frac{2}{\sqrt{5}}$

$y = 4\cos x = 4\left(\frac{1}{\sqrt{5}}\right) = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$

Coordinates of intersection : $\left(\arctan 2, \frac{4\sqrt{5}}{5}\right)$

(iii) A student argues that without the condition $0° \leqslant x \leqslant 180°$all the points of intersection of the graphs would occur at intervals of $360°$ because both $\sin x$ and $\cos x$ are periodic functions with this period. Comment on the validity of the student's argument. **[1]**

iii) The period of $\tan x = 180°$ so their argument is invalid

6 In this question you must show detailed reasoning.

You are given that $f(x) = 4x^3 - 3x + 1$.

(i) Use the factor theorem to show that $(x+1)$ is a factor of $f(x)$. **[2]**

6 i) $f(x) = 4x^3 - 3x + 1$

$f(-1) = -4 + 3 + 1$

$= 0$

hence $(x+1)$ is a factor

(ii) Solve the equation $f(x) = 0$. **[3]**

ii) $f(x) = (x+1)(ax^2 + bx + c)$

$$\begin{array}{r}
4x^2 - 4x + 1 \\
x+1 \,\big)\, \overline{4x^3 - 3x + 1} \\
4x^3 + 4x^2 \\
\hline
-4x^2 - 3x + 1 \\
-4x^2 - 4x \\
\hline
x + 1 \\
x + 1 \\
\hline
0
\end{array}$$

$f(x) = (x+1)(4x^2 - 4x + 1)$

$f(x) = (x+1)(2x-1)^2$

$x = -1, \ x = \frac{1}{2}$

7 A toy boat of mass 1.5 kg is pushed across a pond, starting from rest, for 2.5 seconds. During this time, the boat has an acceleration of $2\,\text{m}\,\text{s}^{-2}$. Subsequently, when the only horizontal force acting on the boat is a constant resistance to motion, the boat travels 10 m before coming to rest. Calculate the magnitude of the resistance to motion.

7. First find the speed the boat reaches after the first 2.5 seconds

$s = -$	
$u = 0$	$v = u + at$
$v = v$	$v = 0 + 2(2.5)$
$a = 2$	$v = 5$
$t = 2.5$	

Now find the deceleration in the second part of it's journey

$s = 10$	
$u = 5$	$v^2 = u^2 + 2as$
$v = 0$	$0 = 5^2 + 2a(10)$
$a = a$	$a = \dfrac{-25}{2 \times 10}$
$t = -$	

$a = -1.25$

$F = ma$

$F = 1.5(-1.25)$

$F = -1.875$

Magnitude = 1.875

8 In this question you must show detailed reasoning.

Fig. 8 shows the graph of a quadratic function. The graph crosses the axes at the points $(-1, 0)$, $(0, -4)$ and $(2, 0)$.

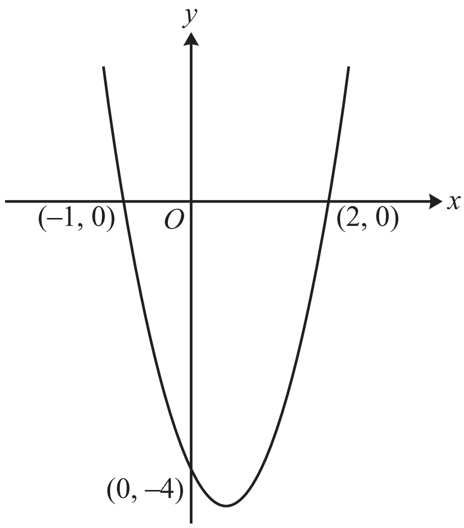


Fig. 8

Find the area of the finite region bounded by the curve and the x-axis. **[8]**

8. The equation has the form $y = k(x+1)(x-2)$

To find k, sub in the coordinates of $(0, -4)$

$-4 = k(1)(-2)$

$-4 = -2k$

$k = 2$

$\therefore \quad y = 2(x+1)(x-2)$

$$\text{Area} = \int_{-1}^{2} 2(x^2 - x - 2) \, dx$$

$$= 2\int_{-1}^{2} x^2 - x - 2 \, dx$$

$= 2\left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\right]_{-1}^{2}$

$= 2\left[\frac{1}{3}(8) - \frac{1}{2}(4) - 2(2) + \frac{1}{3}(1) + \frac{1}{2}(1) - 2\right]$

$= 2\left[\frac{8}{3} - 2 - 4 + \frac{1}{3} + \frac{1}{2} - 2\right]$

$= 2 \times -4.5$

$= -9$

This value is negative because the area is below the x axis. But area has to be positive.

Area $= 9$

9 The curve $y = (x-1)^2$ maps onto the curve C_1 following a stretch scale factor $\frac{1}{2}$ in the x-direction.

(i) Show that the equation of C_1 can be written as $y = 4x^2 - 4x + 1$. **[2]**

The curve C_2 is a translation of $y = 4.25x - x^2$ by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

9 i) let $y = f(x)$

C_1 : $y = f(2x) = (2x-1)^2$

$= 4x^2 - 4x + 1$

[7]

(ii) Show that the normal to the curve C_1 at the point (0, 1) is a tangent to the curve C_2.

ii) The curve has been shifted down by 3

So C_2 is $y = 4.25x - x^2 - 3$

Normal to C_1 : $\frac{dy}{dx} = 8x - 4$

At $x = 0$, $\frac{dy}{dx} = -4$

Intersection of C_2 with $y = \frac{1}{4}x + 1$

$\frac{1}{4}x + 1 = 4.25x - x^2 - 3$

$x + 4 = 17x - 4x^2 - 12$

$4x^2 - 16x + 16 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

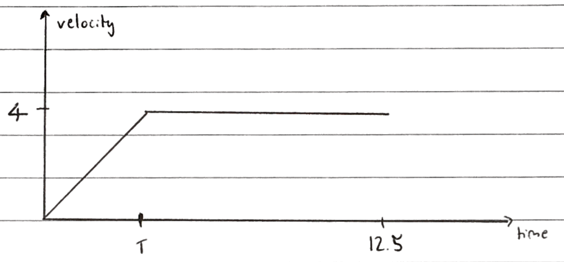
There is only one solution to this equation meaning the line only intersects the curve once. Therefore it is a tangent to C_2

10 Rory runs a distance of 45 m in 12.5 s. He starts from rest and accelerates to a speed of $4\,\text{m}\,\text{s}^{-1}$. He runs the remaining distance at $4\,\text{m}\,\text{s}^{-1}$.

Rory proposes a model in which the acceleration is constant until time T seconds.

(i) Sketch the velocity-time graph for Rory's run using this model. **[2]**

10. i)



(ii) Calculate T. **[2]**

ii) Distance = Area under the curve

$45 = \frac{4 \times T}{2} + (12.5 - T) \times 4$

$45 = 2T + 50 - 4T$

$2T = 5$

$T = 2.5$

[2]

[1]

(iii) Find an expression for Rory's displacement at time t s for $0 \leqslant \leqslant T$. **[2]**

iii) $a = \dfrac{4}{2.5} = 1.6$

$s = s$

$u = 0$ $\quad s = ut + \frac{1}{2}at^2$

$v = -$ $\quad s = 0 + \frac{1}{2}(1.6)t^2$

$a = 1.6$ $\quad s = 0.8t^2$

$t = t$

(iv) Use this model to find the time taken for Rory to run the first 4 m. **[2]**

iv) $0.8t^2 = 4$

$t^2 = 5$

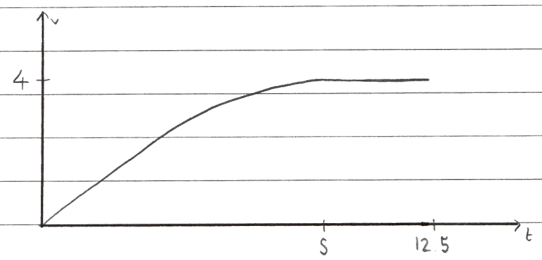
$t = \sqrt{5} = 2.24$

[1]

Rory proposes a refined model in which the velocity during the acceleration phase is a quadratic function of t. The graph of Rory's quadratic goes through (0, 0) and has its maximum point at (S, 4). In this model the acceleration phase lasts until time S seconds, after which the velocity is constant.

(v) Sketch a velocity-time graph that represents Rory's run using this refined model. **[1]**

v)



(vi) State with a reason whether S is greater than T or less than T. (You are not required to calculate the value of S.) **[1]**

vi) The total area under the two graphs can only be the same if $S > T$

11 The intensity of the sun's radiation, y watts per square metre, and the average distance from the sun, x astronomical units, are shown in Fig. 11 for the planets Mercury and Jupiter.

	x	y
Mercury	0.3075	14400
Jupiter	4.950	55.8

Fig. 11

The intensity y is proportional to a power of the distance x.

(i) Write down an equation for y in terms of x and two constants. **[1]**

11. i) $y = kx^n$

(ii) Show that the equation can be written in the form ln y = a+b ln x. **[2]**

ii) $\ln y = \ln kx^n$

$\ln y = \ln k + \ln x^n$

$\ln y = \ln k + n \ln x$

(iii) In the Printed Answer Booklet, complete the table for $\ln x$ and $\ln y$ correct to 4 significant figures. **[2]**

iii)

	$\ln x$	$\ln y$
Mercury	-1.179	9.575
Jupiter	1.599	4.022

(iv) Use the values from part **(iii)** to find a and b. **[3]**

iv) $9.575 = \ln k - 1.179n$ $\quad$ $4.022 = \ln k + 1.599n$

$9.575 = a - 1.179b$ $\quad$ $4.022 = a + 1.599b$

$9.575 + 1.179b = 4.022 - 1.599b$

$2.778b = -5.553$

$b = -1.999$

$a = 9.575 + 1.179(-1.999)$

$a = 7.218$

(v) Hence rewrite your equation from part **(i)** for y in terms of x, using suitable numerical values for the constants. **[2]**

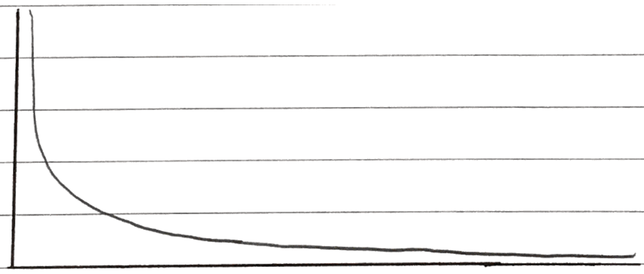
v) $\ln k = a = 7.218$ $\quad\quad n = b = -1.999$

$k = 1364$

$y = 1364x^{-1.999}$

(vi) Sketch a graph of the equation found in part **(v)**. **[2]**

vi)



(vii) Earth is 1 astronomical unit from the sun. Find the intensity of the sun's radiation for Earth. **[1]**

vii) $x = 1, \quad y = 1364(1) = 1364 \text{ Wm}^{-2}$

END OF QUESTION PAPER

BLANK PAGE